# Minimum Variance Portfolio Construction

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### **Presentation Outline**

- 1. Introduction
- 2. Method
- 3. Empirical Analysis and Results
- 4. Beta Analysis in Specific Market Conditions



### 1. Introduction

- What is a Minimum Variance Portfolio (MVP)?
- Why Russell 3000?
- Comparison of long-only vs. long-short portfolio
- This presentation follows the methodology and findings of Clarke, de Silva, and Thorley (2011).



### Minimum Variance Portfolio Definition

$$\begin{split} \min_{\mathbf{w}} & \mathbf{w}^\top \Omega \, \mathbf{w} \\ \text{subject to} & \mathbf{1}^\top \mathbf{w} = 1, \\ & \mathbf{w} \geq 0 \quad (\text{long-only constraint}) \end{split}$$

Where:

- w is the vector of asset weights,
- $\Omega$  is the covariance matrix of asset returns



### 2. Method

- Covariance Estimation and Variance Calculation
- Beta and Idiosyncratic Variance Estimation
- Long-Short (LS) Portfolio Construction
- Long-Only (LO) Portfolio Construction



# Covariance Estimation and Market Variance

- Daily stock returns used to estimate the covariance matrix.
- Ledoit–Wolf shrinkage improves accuracy by shrinking the sample covariance toward a structured target.
- Particularly effective in high-dimensional settings (3000 stocks).

Market Variance:

$$\sigma_{\rm M}^2 = {\rm w}_{\rm M}^{\rm T} \Omega {\rm w}_{\rm M}$$

- $\Omega$ : shrunk covariance matrix.
- *w<sub>M</sub>*: market-cap weight vector.



# One-Factor Model, Beta, and Idiosyncratic Variance

Single-Factor Model:

$$R_i = \alpha_i + \beta_i R_M + \epsilon_i$$

**Beta Estimation:** 

$$\beta_i = \frac{(\Omega w_M)_i}{\sigma_M^2}$$

#### **Idiosyncratic Variance:**

$$\sigma_{\epsilon,i}^2 = \Omega_{ii} - \beta_i \sigma_M^2$$

- $\alpha_i$ : asset-specific intercept (alpha)
- $\beta_i$ : sensitivity to market
- +  $\sigma^2_{\epsilon,i}$ : variance not explained by market



### Long-Short (LS) Portfolio

#### Beta Threshold $\beta_{LS}$ :

$$\beta_{\text{LS}} = \frac{\frac{1}{\sigma_{\text{M}}^2} + \sum \left(\frac{\beta_i^2}{\sigma_{\epsilon,i}^2}\right)}{\sum \left(\frac{\beta_i}{\sigma_{\epsilon,i}^2}\right)}$$

#### Weight Assignment:

$$\mathbf{w}_{i} = \frac{\sigma_{MV}^{2}}{\sigma_{\epsilon,i}^{2}} \left(1 - \frac{\beta_{i}}{\beta_{LS}}\right)$$

• Allows both positive and negative weights.

Methodology and findings of Clarke, de Silva, and Thorley (2011).



# Long-Only (LO) Portfolio

Beta Threshold  $\beta_L$ :

$$\beta_{L} = \frac{\frac{1}{\sigma_{M}^{2}} + \sum_{i=1}^{k} \left(\frac{\beta_{i}^{2}}{\sigma_{\epsilon,i}^{2}}\right)}{\sum_{i=1}^{k} \left(\frac{\beta_{i}}{\sigma_{\epsilon,i}^{2}}\right)}$$

• Computed using the *k* selected stocks with  $\beta_i \leq \beta_L$ .

#### Weight Assignment:

$$w_i = \frac{\sigma_{MV}^2}{\sigma_{\epsilon,i}^2} \left(1 - \frac{\beta_i}{\beta_L}\right), \quad i = 1, \dots, k$$

• Stocks with  $\beta_i > \beta_L$  are assigned weights of zero ( $w_i = 0$ ).

Methodology and findings of Clarke, de Silva, and Thorley (2011).



#### **Threshold Filtering: Calculations and Mistakes**

Beta	Cumulative $eta_{\it L}$	Valid Sum?
-1.148	-49.817	False
-0.963	-36.584	False
÷	÷	÷
0.084	-257.916	False
0.085	113.806	True
0.087	78.486	True
÷	÷	÷
0.521	0.522	True
0.521	0.522	True
0.521	0.522	True
0.522	0.522	False
0.522	0.522	False



#### Filtered Stocks with Magic R<sub>i</sub> to get Weight

	Beta	Cumulative $eta_{\it L}$	Valid Sum?	R <sub>i</sub>	Valid R <sub>i</sub> ?	Weight	
	-1.148	-49.817	False	12205.733	True	0.005	
	-0.963	-36.584	False	12252.104	True	0.002	
	÷	÷	:	÷	:	÷	
	0.084	-257.916	False	12972.067	True	0.010	
	0.085	113.806	True	12972.124	True	0.011	
	0.087	78.486	True	12971.894	True	0.003	
	÷	÷	÷	÷	÷	÷	
	0.521	0.522	True	130.408	True	0.001	
	0.521	0.522	True	109.244	True	0.001	
	0.521	0.522	True	0.456	True	0.001	
	0.522	0.522	False	-5.884	False	0.000	
Re	0.522	0.522	False	-28.840	False	0.000	
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# 3. Empirical Analysis and Results

- Performance Comparison: MVP Long-Only, MVP Long-Short, and Market Portfolio.
- Rebalancing and Beta Ranges
- Diversification and Exposure Risk







Beta Ranges at Rebalance Dates





### **Diversification and Exposure Risk**



**Effective Number of Stocks** 

Long Exposure for LS Portfolio



#### 4. Beta Analysis in Specific Market Conditions

- Why These Periods?
  - Turbulent Market (H2 2022) vs. Exuberant Market (H2 2020)
- Contextual Background:
  - Market Indices over these periods for context
- Beta and Volatility Analysis:
  - Box plots of betas and annualized volatility
- Volatility vs. Beta Relationship:
  - Visualizing beta thresholds
- Weights and Risk:
  - Security weights vs. betas
  - Security weights vs. idiosyncratic risk



### Market Context



Exuberant Market H2 2020

Turbulent Market H2 2022



### Beta Box Plots



Exuberant Market H2 2020

#### Turbulent Market H2 2022



# Annualized Specific Volatility



#### Exuberant Market H2 2020

#### Turbulent Market H2 2022



# Volatility vs. Betas



Exuberant Market H2 2020

 $\beta_{LS} = 1.095$  $\beta_{l} = 0.522$ 

#### Turbulent Market H2 2022

 $\beta_{\rm LS} = 1.141$  $\beta_{\rm L} = 0.392$ 



### Security Weight vs. Betas



Exuberant Market H2 2020

#### Turbulent Market H2 2022



# Security Weight vs. Idiosyncratic Risk



Exuberant Market H2 2020

#### Turbulent Market H2 2022



### Top 10 Holdings: Long-Only vs. Long-Short (Long Side) 2020

#### **Long-Only Portfolio**

Holding	Weight
JUNIPER INDUSTRIAL HOLDINGS INC	1.35%
OSPREY TECHNOLOGY ACQ CORP	1.33%
UNION ACQUISITION CORP II	1.30%
CRESCENT ACQUISITION CORP	1.28%
EAST STONE ACQUISITION CORP	1.26%
ALUSSA ENERGY ACQUISITION CORP	1.25%
TIFFANY & CO NEW	1.25%
CHINA BIOLOGIC PRODUCTS HDS INC	1.24%
CITIC CAPITAL ACQUISITION CORP	1.24%
S C V X CORP	1.22%

#### Long-Short Portfolio (Long Side)

Holding	Weight
OSPREY TECHNOLOGY ACQ CORP	0.45%
UNION ACQUISITION CORP II	0.45%
CRESCENT ACQUISITION CORP	0.45%
JUNIPER INDUSTRIAL HOLDINGS INC	0.45%
EAST STONE ACQUISITION CORP	0.45%
ALUSSA ENERGY ACQUISITION CORP	0.44%
CHINA BIOLOGIC PRODUCTS HDS INC	0.43%
CINCINNATI BELL INC NEW	0.43%
GALILEO ACQUISITION CORP	0.43%
CITIC CAPITAL ACQUISITION CORP	0.43%



#### Top 5 Shorted Stocks in Long-Short Portfolio 2020

Holding	Weight
APPLE INC	-0.27%
MICROSOFT CORP	-0.21%
ADVANCED ENERGY INDUSTRIES INC	-0.19%
TESLA INC	-0.19%
STMICROELECTRONICS NV	-0.18%



### Thank You for Your Interest

#### Passing the baton to Ololade!

