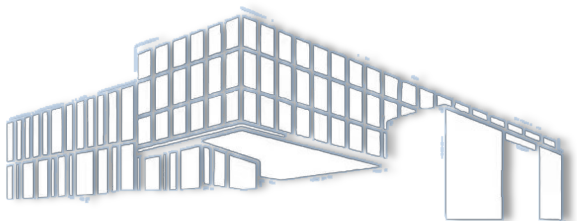
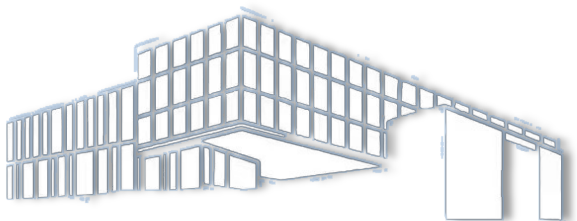
Outline

- Introduction of the question
- From classical statistics to random matrix theories
- Marchenko-Pastur Law
- Duality in MP distribution
- Prospect



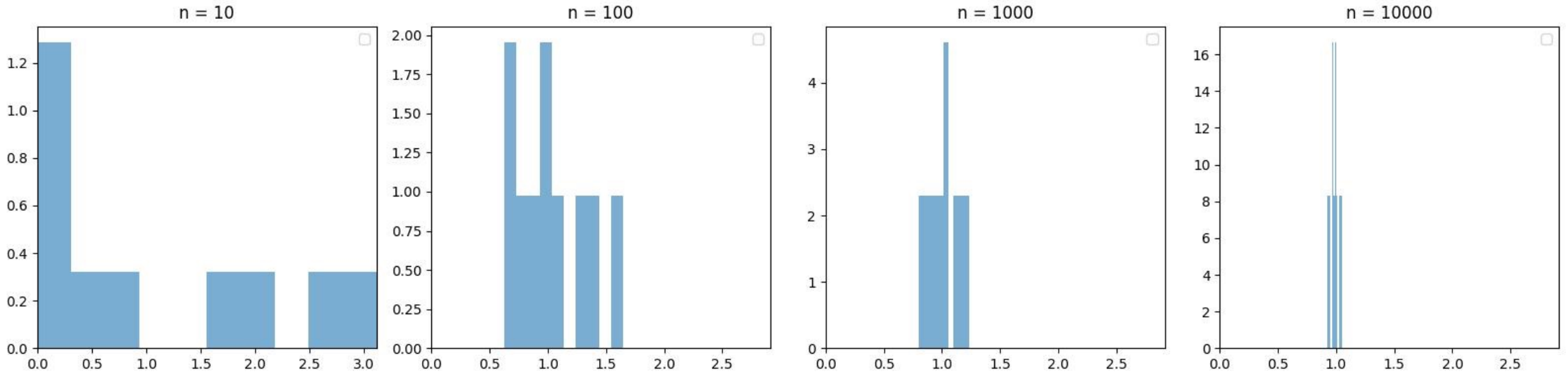
Leading in

- There should be differences between the S&P 500 covariance matrix and a random matrix (generated by n observations of p-dimensional standard normal)
- And we know much more about random matrices
- Question: How does the spectrum of eigenvalues depend on p and n?

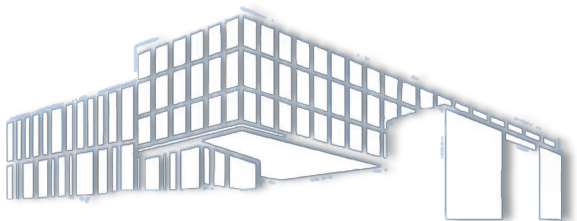


Classical statistics

- p is constant (e.g. $p = 10$), $n \to \infty$



- But things change when p and n go to infinity together…

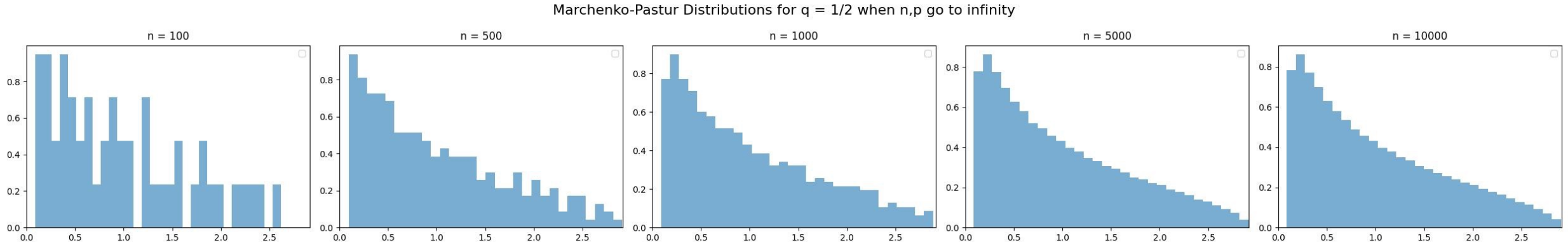


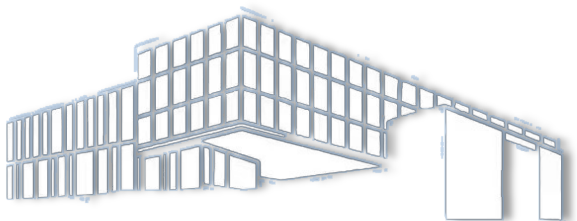
The Marchenko-Pastur Law

- When $p \to \infty$, $n \to \infty$, we assume $q = p/n$ tends to a finite ratio

$$\rho_{MP}(\lambda) = \begin{cases} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{2\pi q \sigma^2}, \text{if } \lambda \in [\lambda_+, \lambda_-] \\ 0, \text{if } \lambda \notin [\lambda_+, \lambda_-] \end{cases}$$

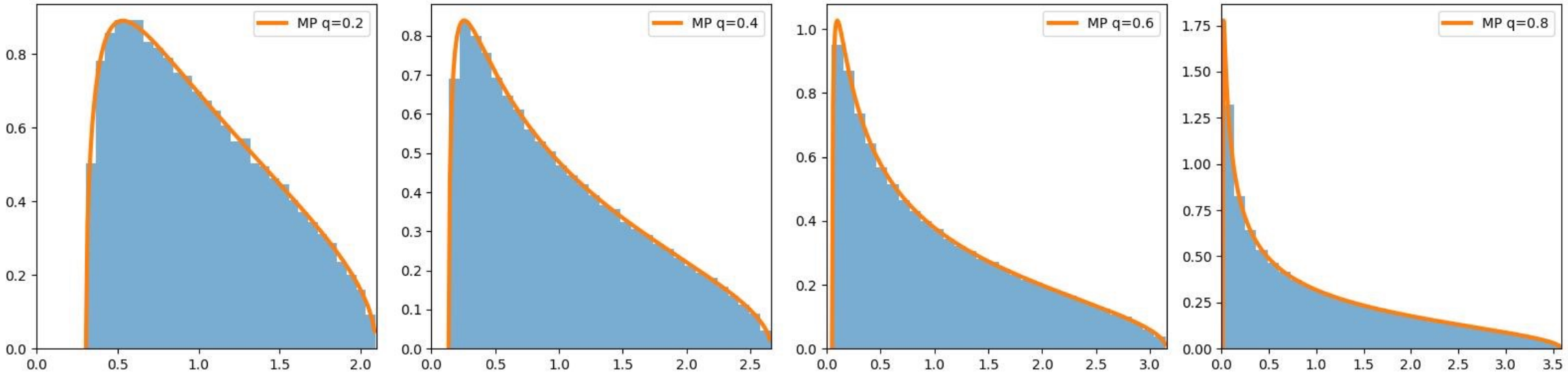
$$\lambda_- = \sigma^2 \left(1 - \sqrt{q}\right)^2 \qquad \lambda_+ = \sigma^2 \left(1 + \sqrt{q}\right)^2$$





Taking different values of q

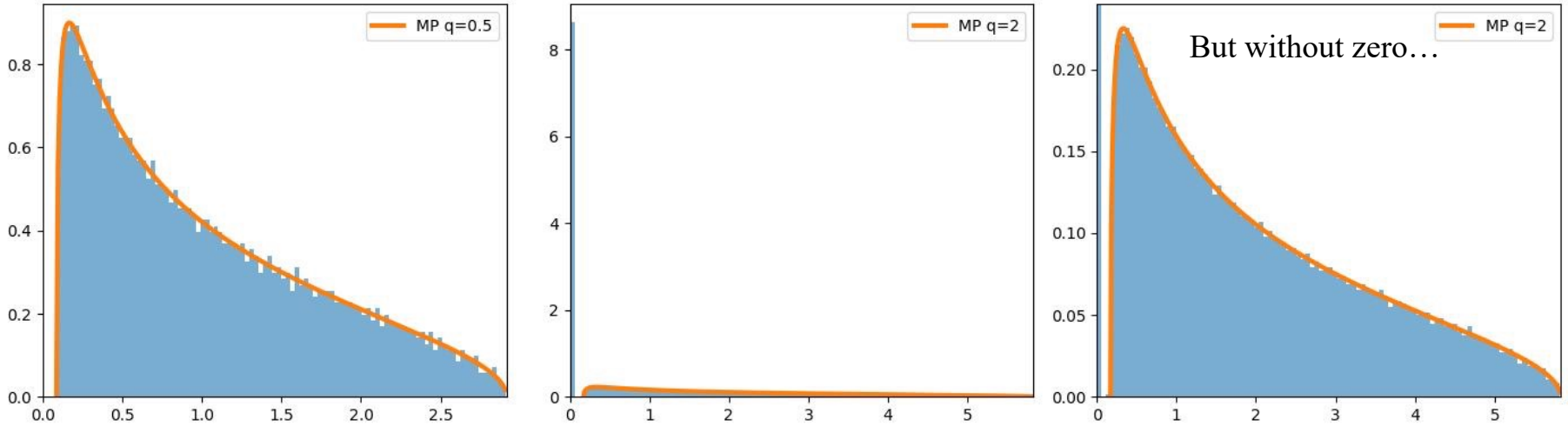
- Normally, we are talking about $q < 1$



- But what if q is larger than one?

Duality in MP

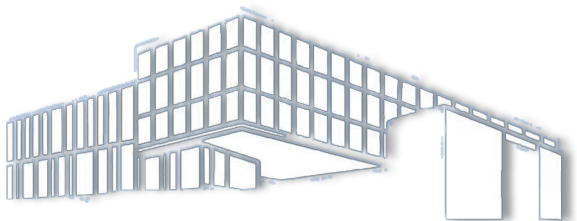
- What happens when $q > 1$ (e.g. q=2 compared to q=1/2)



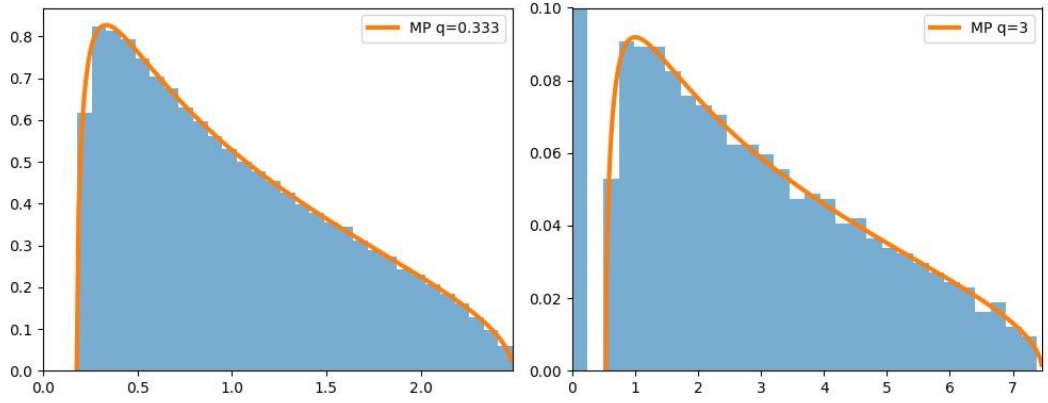
- Duality: Symmetric behavior between MP distribution of q and $1/q$

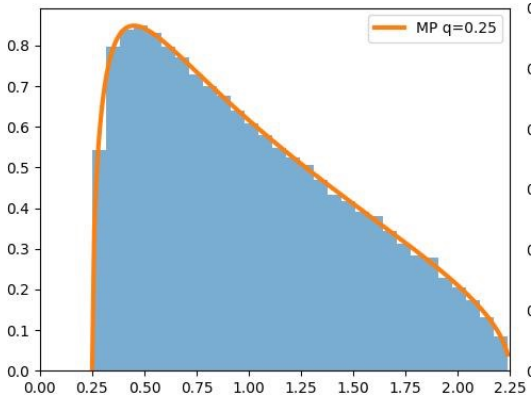
The math behind it

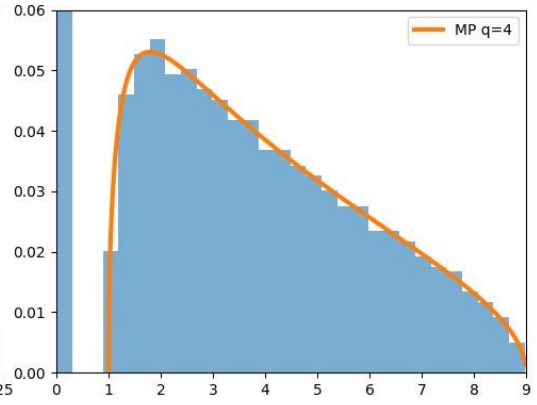
- An intuition from $C_1=XX^T$ and $C_2=X^TX$
- But slightly different…

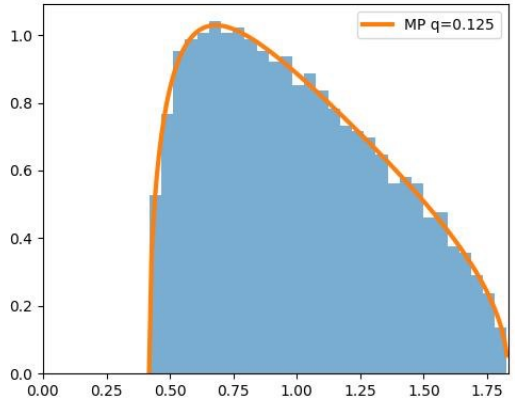


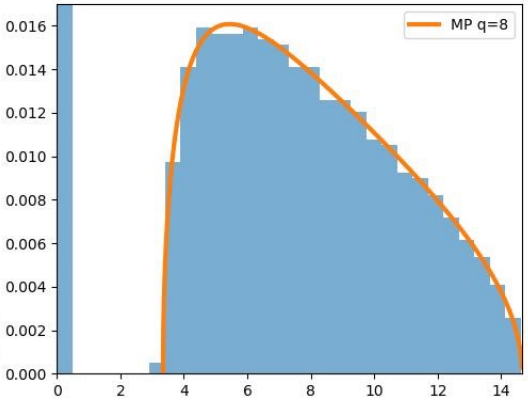
More simulations on duality

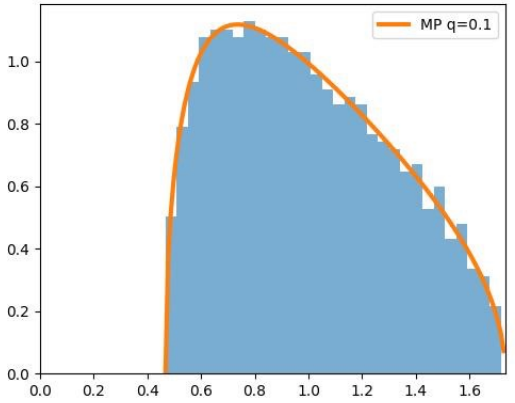


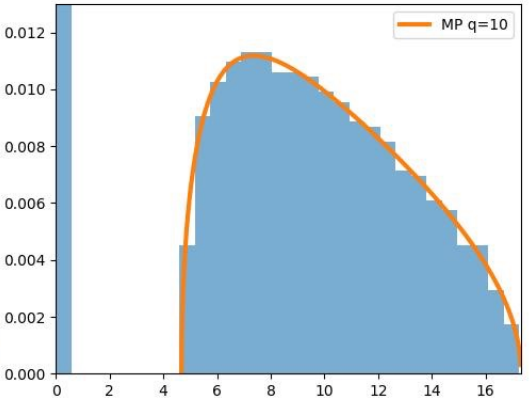


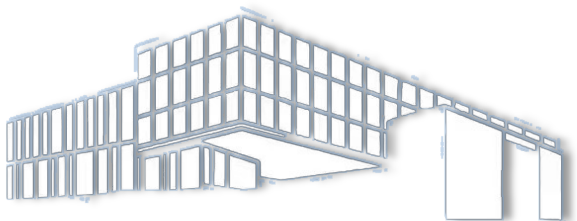












Prospects

- Used in factor models
- Denoising financial data
- And more…